PROFILOMETRY FOR THE LOWER TERRESTRIAL ATMOSPHERE

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Summary : The principle of an Optical Turbulence Profiler based on Angle-of-Arrival statistics is presented. Similar to a SLODAR it is well-adapted to study the terrestrial atmosphere boundary-layer in daytime and nighttime conditions.

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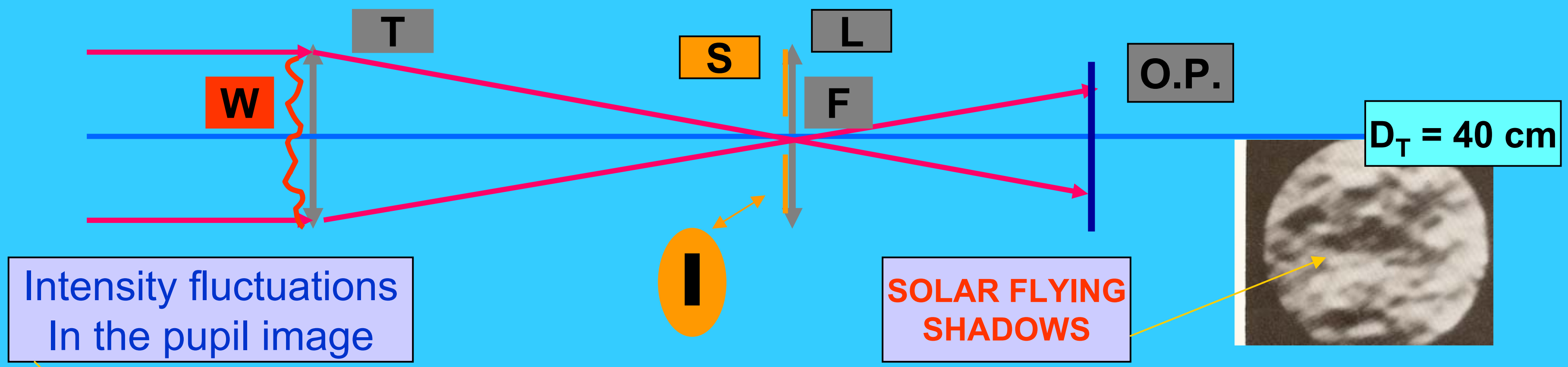
WAVEFRONT ANALYSIS

The telescope pupil is observed through a thin slit placed on the solar (eventually lunar) limb. **At the first order,** one observes intensity fluctuations **proportional to angle-of-arrival fluctuations (indeed fluctuations of the component β(x,y) considered in the direction perpendicular to the solar limb).**

f_T = Telescope T focal length - f_L = Lens L focal length



$$\tilde{I}(x,y) = C\frac{f_T}{f_L}\left[\frac{\lambda f_T}{f_L}\right]^2 \left|\hat{t}\left(\frac{x}{\lambda f_L},\frac{y}{\lambda f_L}\right)\right|^2 * P\left(-x\frac{f_T}{f_L},-y\frac{f_T}{f_L}\right)\left[\beta\left(-x\frac{f_T}{f_L},-y\frac{f_T}{f_L}\right)\right]$$

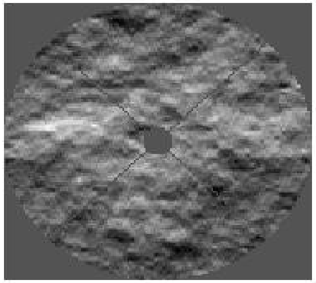
OBSERVATION OF ANGLE-OF-ARRIVAL FLUCTUATIONS : NUMERICAL SIMULATION RESULTS

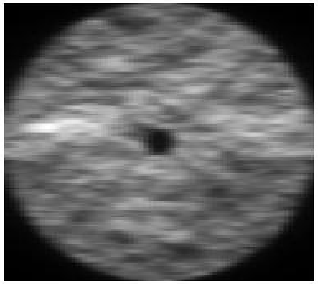
Left : component β(x,y) of angle-of-arrival fluctuations (perpendicular to the solar limb) observed at the level of the telescope entrance pupil.

Right : component β(x,y) in the image of the telescope pupil observed through a thin slit (6 arcseconds width) placed on the solar limb image.

One notes the filtering performs by the slit (diffraction and angular integration)

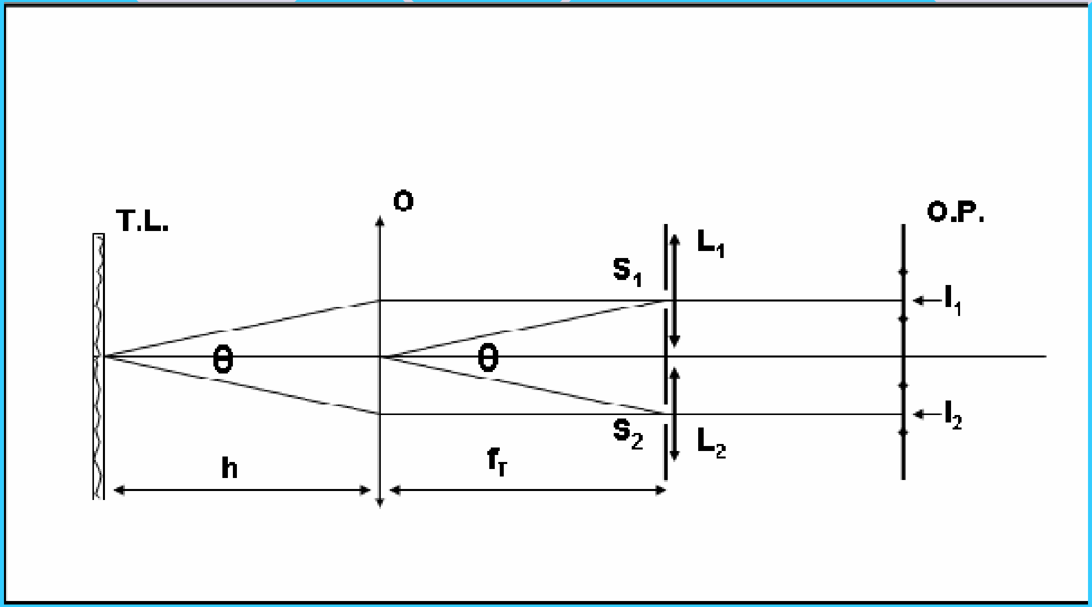
r0=4cm – L0=10m – h=0 – D=30cm (the von Kàrmàn model is assumed)





The validity of this first order approximation may be established using solar limb-darkening models [Van't Veer (1960), Klinglesmith et al. (1970), Diaz-Cordovés et al. (1992), Van Hamme (1993), Hestroffer et al. (1998)]

OPTICAL TURBULENCE PROFILER



The telescope pupil is observed through 2 slits with an angular separation equal to θ. In each direction is obtained a map of the angle-of-arrival component β(x,y), considered in the direction perpendicular to the solar limb. Spatial cross-correlations lead to the vertical distributions of optical turbulence energy $C_n^2(h)$. The angular separation between the 2 slits may be easily changed and thus the vertical resolution and the maximum sensing altitude.

T.L. = turbulent layer
O = telescope
f_T = telescope focal length
S_1, S_2 = slits
L_1,L_2 = lenses
O.P. = observation plane
I1,I2 = images

This is a well-known triangulation method.

Transverse Angle-of-Arrival spatial covariance (I) : modelisation

The general expression of this covariance writes as (here Fresnel diffraction has been neglected):

$$C_{\beta}(b) = \pi\lambda^{2}\int_{0}^{+\infty} df\ .f^{3}.W_{\varphi}(f)\left[J_{0}(2\pi bf) + J_{2}(2\pi bf)\right]\left[\frac{2J_{1}(\pi Df)}{\pi Df}\right]^{2}$$

where $W\varphi(f)$ is the power spectrum of the phase fluctuations which is, in the case of a multi-layered turbulent atmosphere and if the inner scale is assumed equal to 0 :

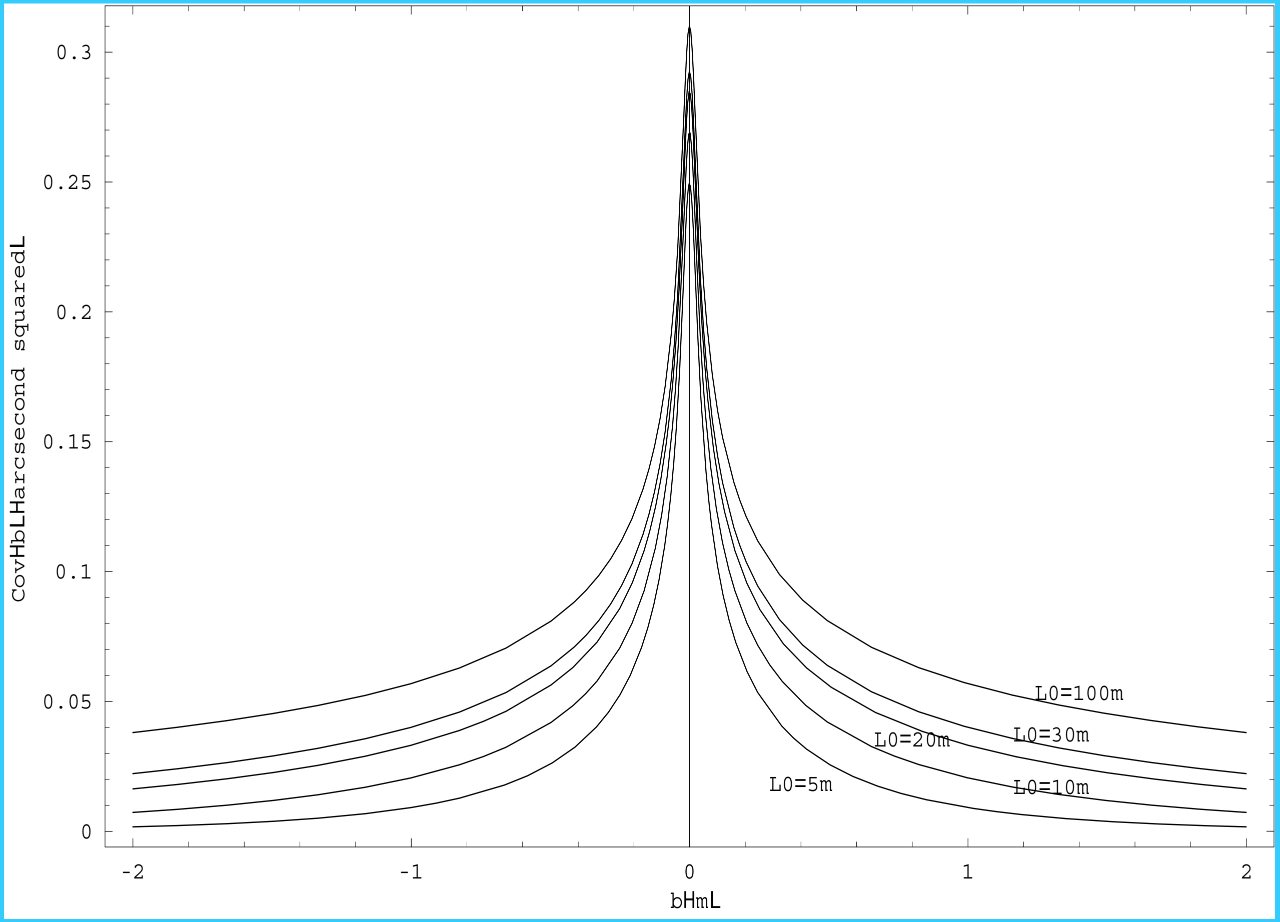
$$W_{\varphi}(\vec{f}) = 0.38\lambda^{-2}\sum_{j} C_{n}^{2}(h_{j})\delta h_{j}\Phi(f, L_{0}(h_{j}))$$

with $\Phi(f, L_0(h_j)) = (f^2 + L_0(h_j)^{-2})^{-11/6}$ (von Kàrmàn model)

or $\Phi(f, L_0(h_j)) = (f^2 + fL_0(h_j)^{-1})^{-11/6}$ (Greenwood-Tarazano model)

or $\Phi(f, L_0(h_j)) = f^{-11/3}(1 - \exp(-f^2 L_0(h_j)^2))$ (exponential model)

Transverse Angle-of-Arrival spatial covariance (II) : effect of the outer scale (von Kàrmàn model)

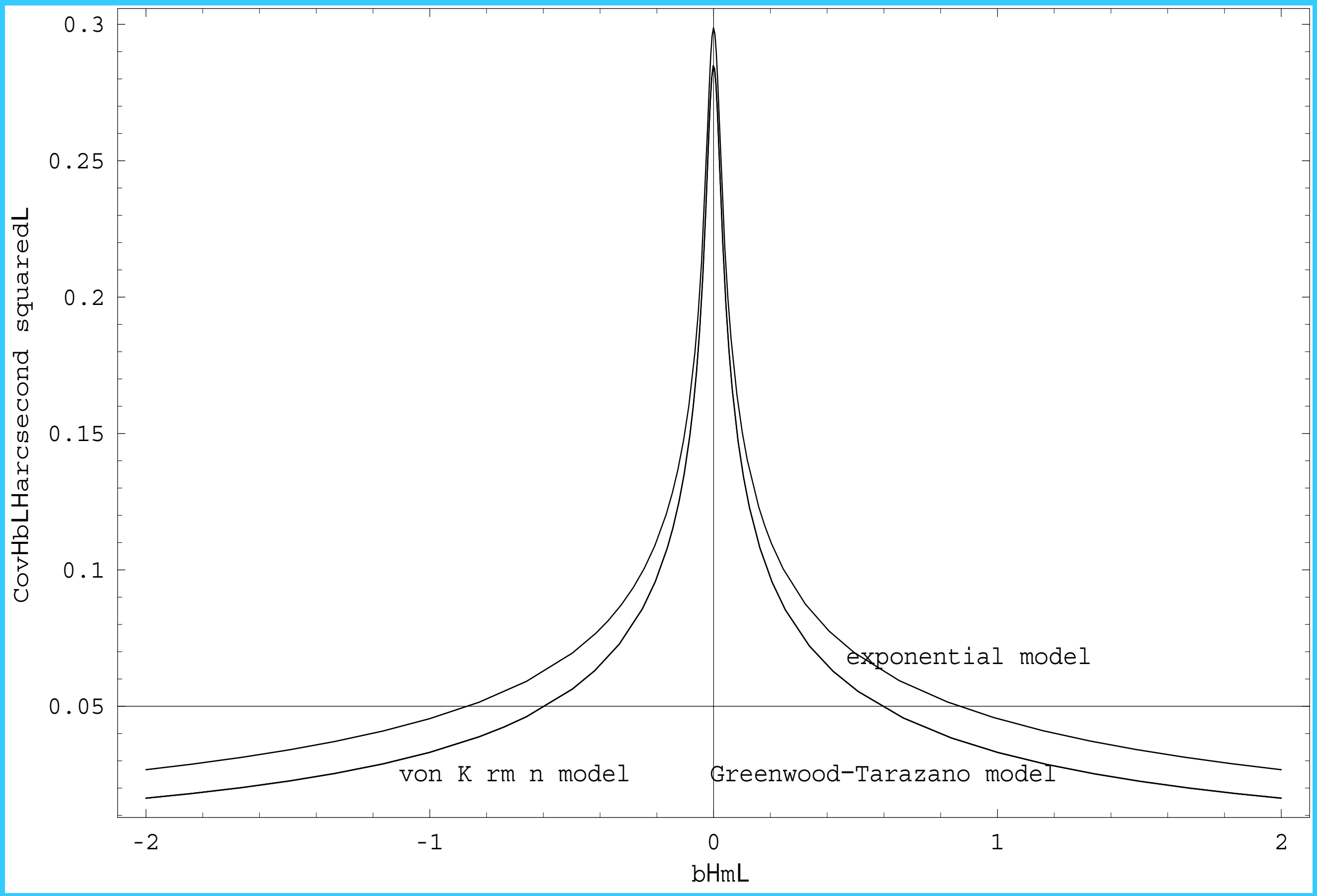


This covariance is drawn with :
λ = 468nm
r_0 = 6cm
Δd = 3cm
$\Delta\theta$ = 3as

The outer scale L_0 varies from 5 to 100m.

Smaller is the outer scale, higher is the altitude resolution.

Transverse Angle-of-Arrival spatial covariance (III) : effect of the turbulence model



This covariance is drawn with :
λ = 468nm
r_0 = 6cm
Δd = 3cm
$\Delta\theta$ = 3as
L_0 = 20m

The altitude resolution is higher in the cases of von Kàrmàn and Greenwood-Tarazano models.

Spatio-angular covariance :study of an hypothetical 4-layer profile [h<1km]

The calculation is performed with turbulence localized in 4 layers at the altitudes :

h1 = 0 ; h2 = 100m ; h3 = 500m ; h3 = 800m

(with the respective weigths 0.60, 0.25, 0.10 and 0.05).

These layers are assumed to represent here 75% of the total optical turbulence energy.

The wavelength is **λ = 468nm ; r_0= 6cm ($\Sigma_j C_n^2(h_j)\,\delta h_j = 1.43\ 10^{-12}\ m^{1/3}$).**

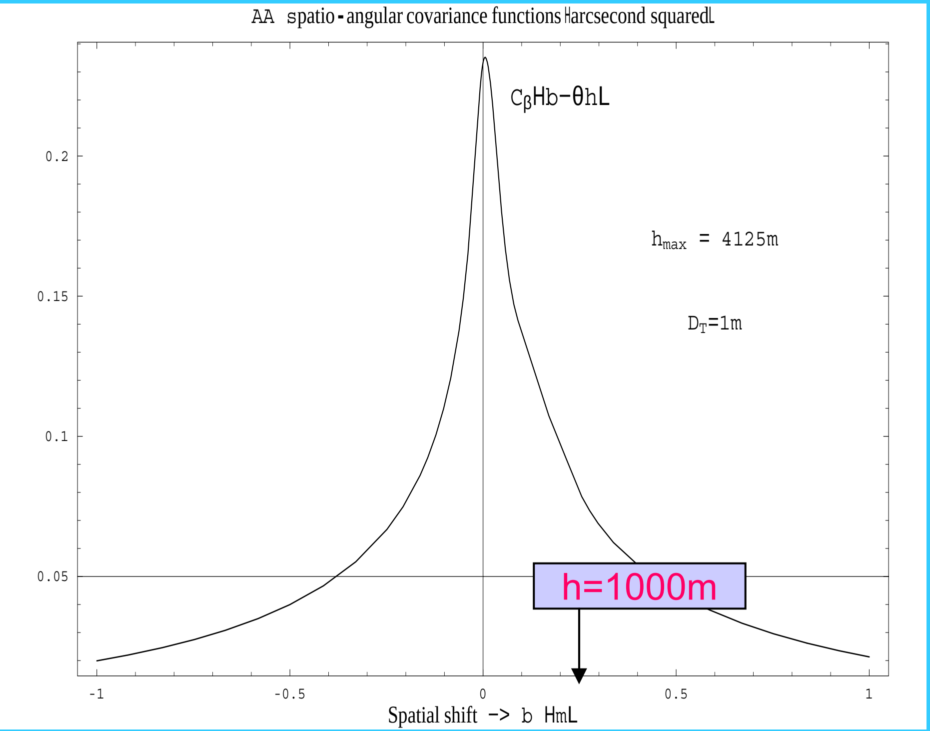
This leads with the above assumptions to : $C_n^2(h_1)\,\delta h_1 = 6.43\ 10^{-13}\ m^{1/3}$, $C_n^2(h_2)\,\delta h_2 = 2.68\ 10^{-13}\ m^{1/3}$, $C_n^2(h_3)\,\delta h_3 = 1.07\ 10^{-13}\ m^{1/3}$, $C_n^2(h_4)\,\delta h_4 = 5.36\ 10^{-14} m^{1/3}$.

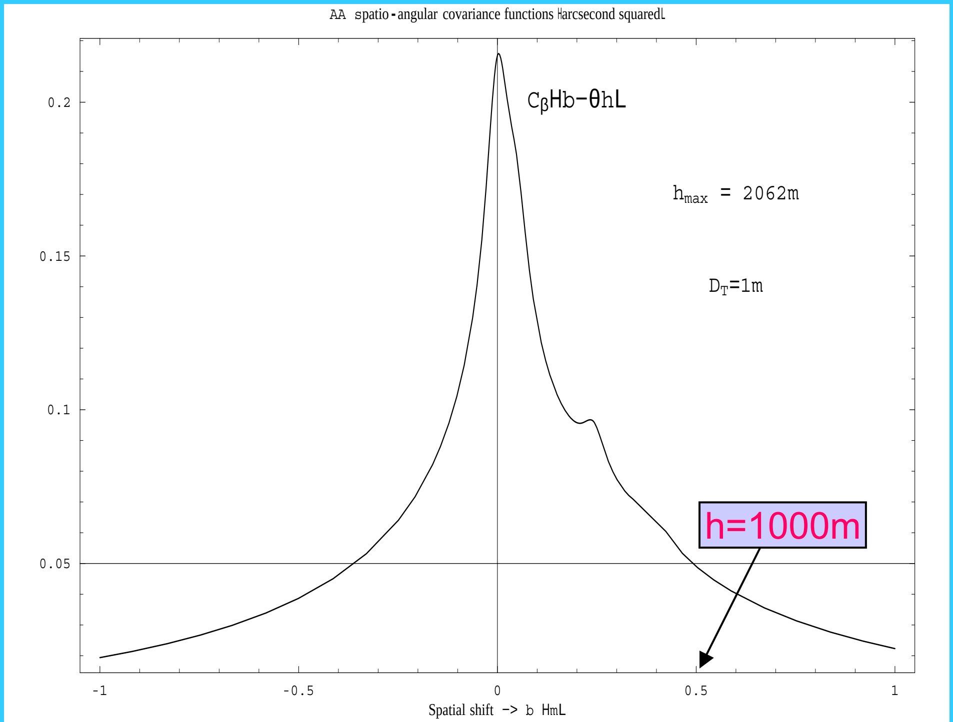
L_0= 10m (supposed constant with the altitude) .

The telescope diameter is **D_T = 1m. The slit width is $\Delta\theta$ = 3 arcseconds.**

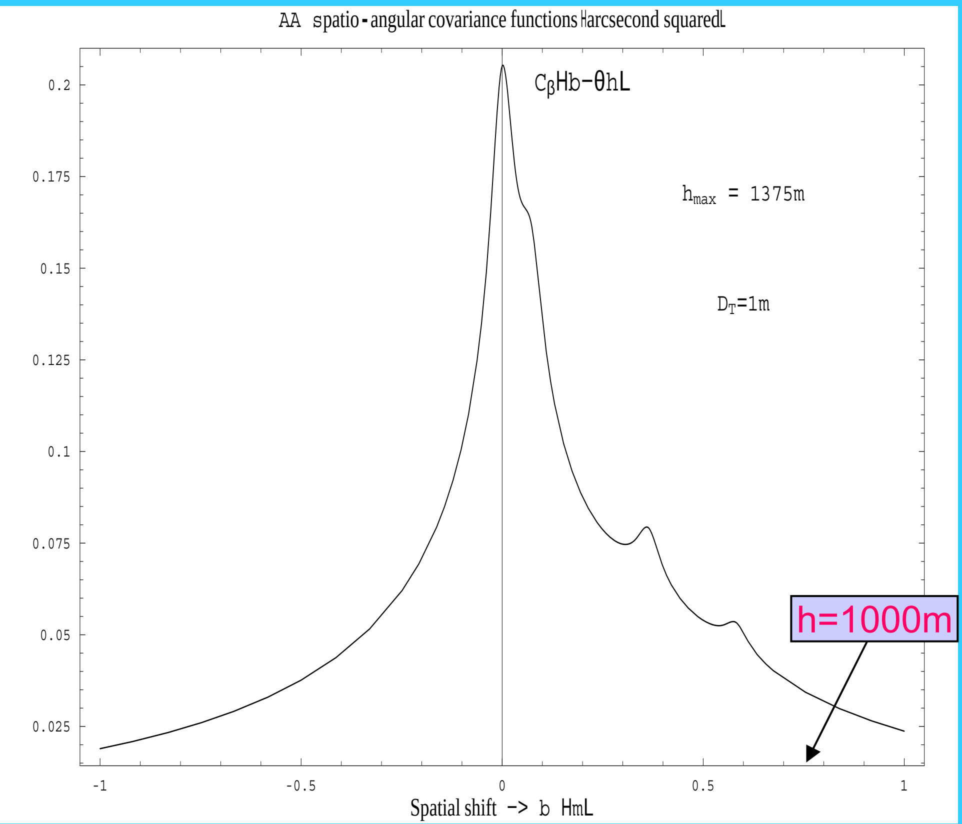
The von Kàrmàn model is assumed.

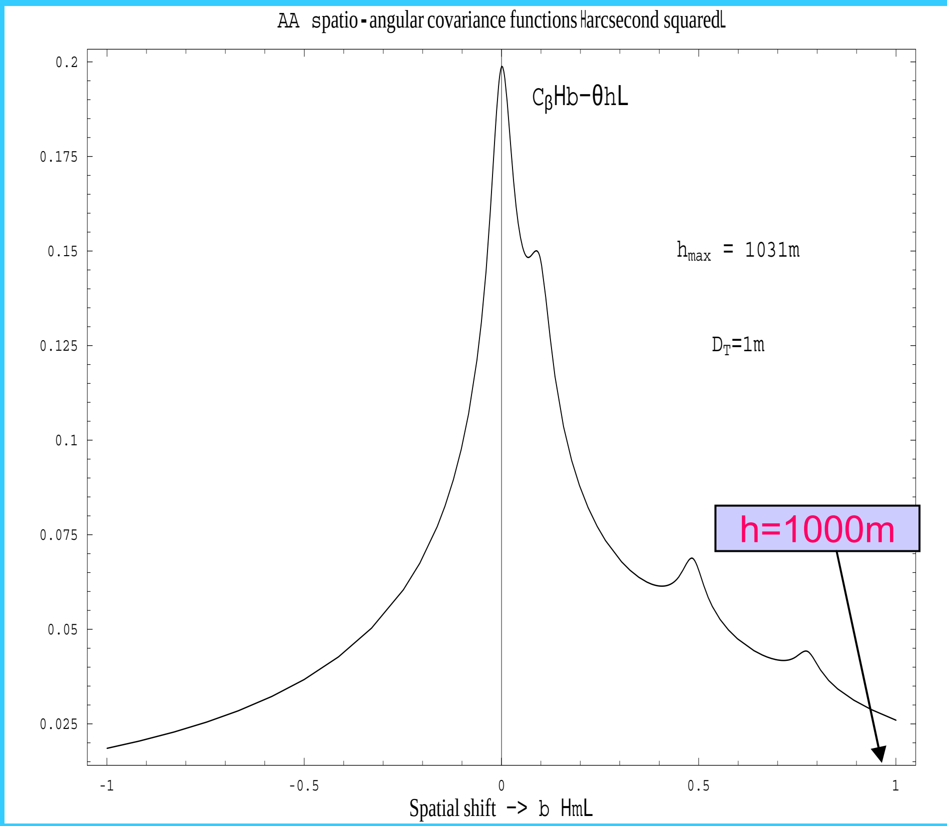
ANGULAR SEPARATIONS = 50 arcseconds (left) and 100 arcseconds (right)



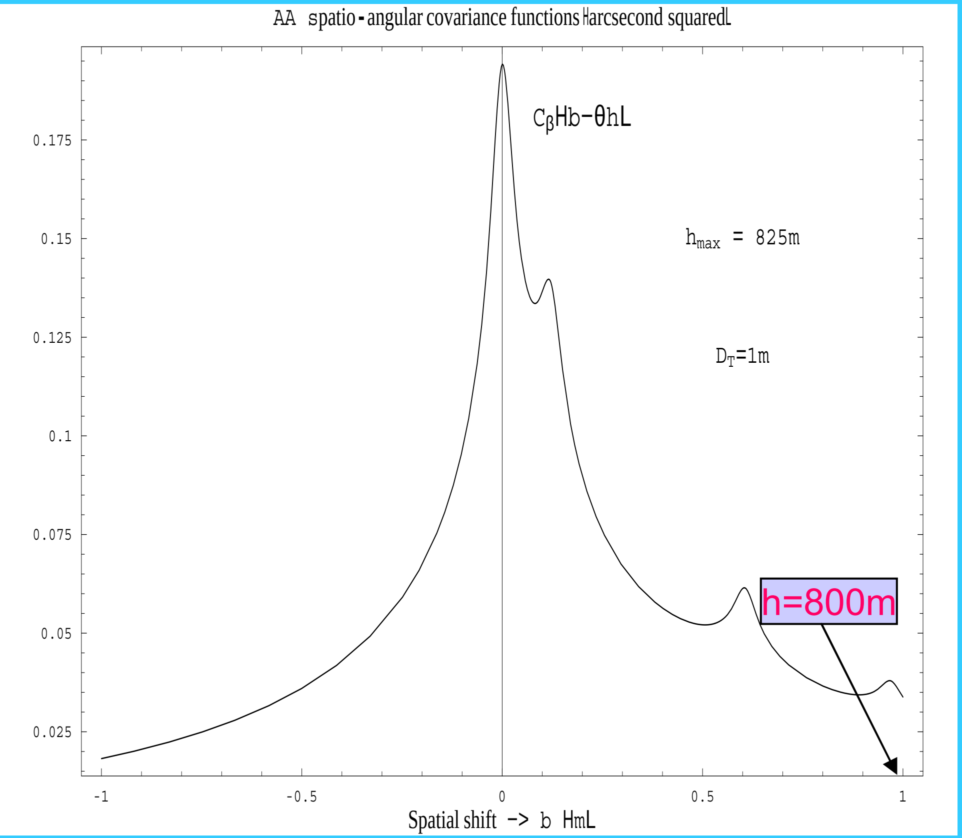


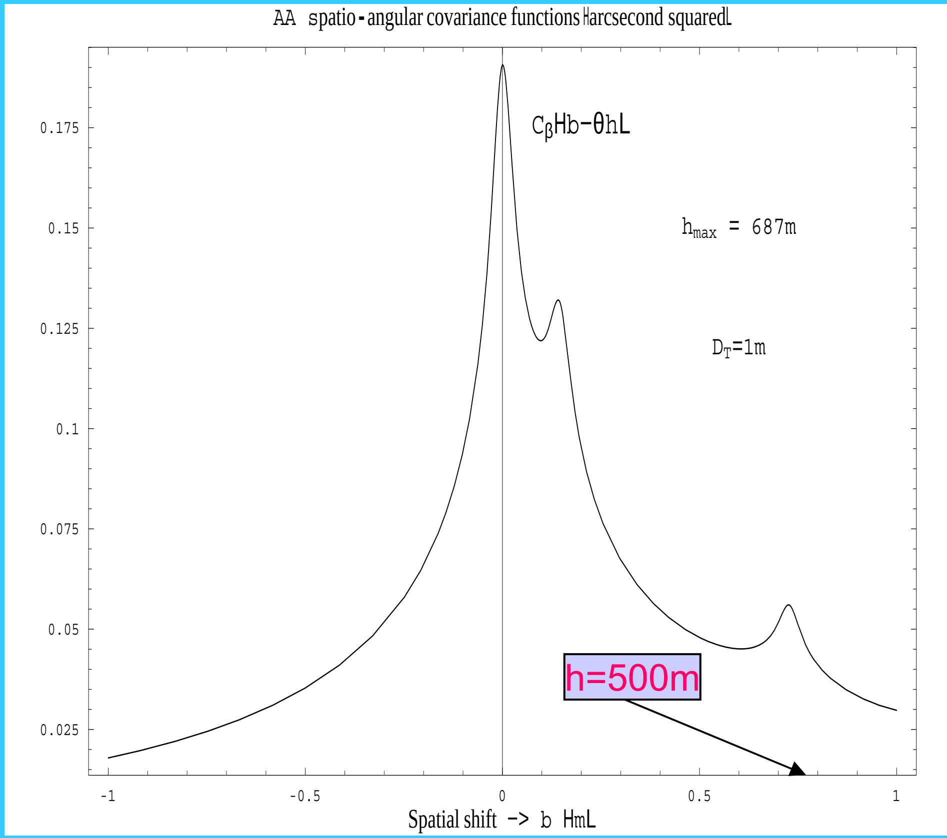
ANGULAR SEPARATIONS = 150 arcseconds (left) and 200 arcseconds (right)





ANGULAR SEPARATIONS = 250 arcseconds (left) and 300 arcseconds (right)





RESTORATION OF THE C_n^2 PROFILE

In the case of a multi-layered turbulence, the transverse spatio-angular covariance of $\beta(x,y)$ may be expressed (von Kàrmàn model) by :

$$C_{\beta,\theta}(b) = \int_0^{+\infty} C_n^{\,2}(h) F(h,\theta,b)\,dh$$

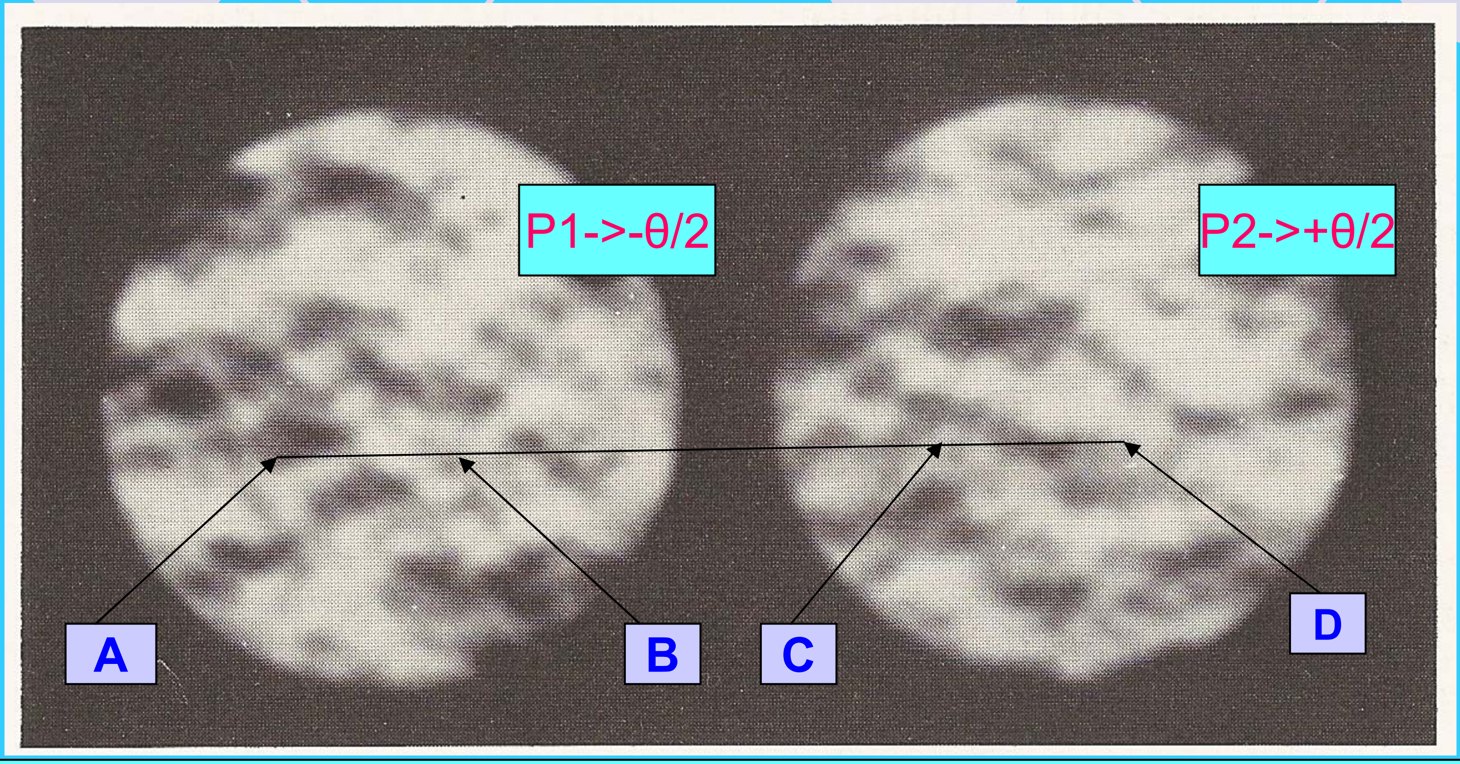
where

$$F(h,\theta,b) = \int_0^{+\infty} df\,.\,f^{\,3}\left(f^{\,2} + \frac{1}{L_0(h)^2}\right)^{-11/6}\left(J_0\left(2\pi f(\theta h - b)\right) + J_2\left(2\pi f(\theta h - b)\right)\right)\left[2\,\frac{J_1(\pi.\Delta d.f)}{\pi.\Delta d.f}\right]^2$$

Retrieving $C_n^2(h)$ (and eventually $L_0(h)$) from $C_{\beta,\theta}(b)$ is a non-linear inverse problem. As that is performed in the case of the MOSP (Monitor of Outer Scale Profile), one can use simulated annealing algorithm for minimizing the cost function E, defined as :

$$E = \sum_b \left[C_{\beta,\theta}^{mes}(b) - C_{\beta,\theta}^{theo}(b)\right]^2$$

DIFFERENTIAL ESTIMATION (I)



P1 and P2 are 2 images of the telescope pupil obtained through 2 diaphragms placed on the solar limb (at the telescope focus) with an angular separation θ = 180 arcseconds.
In each image are observed Solar Flying Shadows.

DIFFERENTIAL ESTIMATION (II)

The angle-of-arrival fluctuations are observed respectively :

At the point A : $\beta_1(x,y,-\theta/2)$ At the point B : $\beta_1(x+b,y,-\theta/2)$

At the point C : $\beta_2(x,y,+\theta/2)$ At the point D : $\beta_2(x+b,y,+\theta/2)$

The spatio-angular covariance writes as :

$$\langle[\beta_1(x,y,-\theta/2) - \beta_2(x,y,+\theta/2)][\beta_1(x+b,y,-\theta/2) - \beta_2(x+b,y,+\theta/2)]\rangle$$

which leads to :

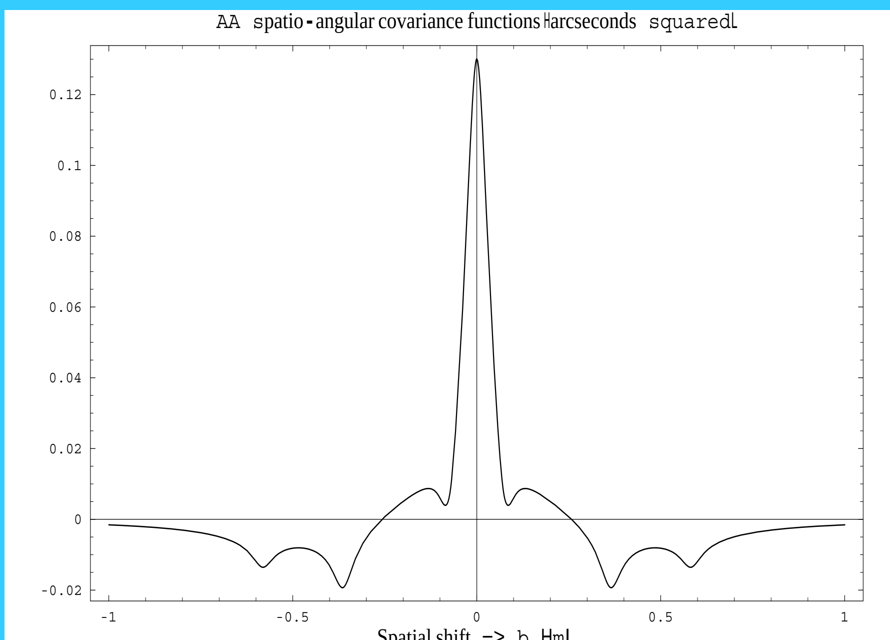
$$CC(b) = C_\beta(b) + C_\beta(b) - C_\beta(b + \theta h) - C_\beta(b - \theta h)$$

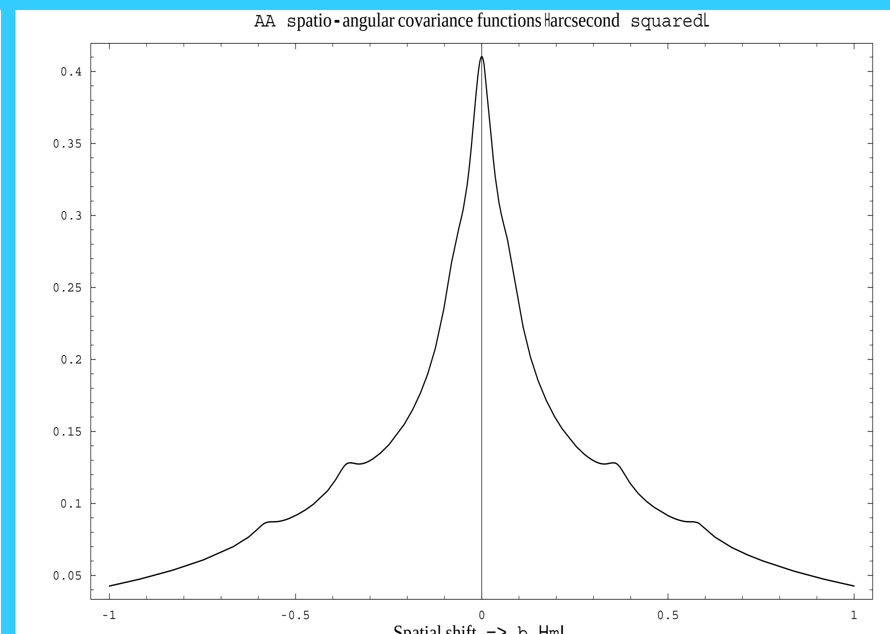
where C_β is the unidimensional covariance, b is a spatial shift on the pupil image and h is the altitude of the turbulent layers.

Representations of CC(b) (left) and $C_\beta(b+\theta h) + C_\beta(b-\theta h)$ (right) [same profile that above with θ = 180 arcseconds]

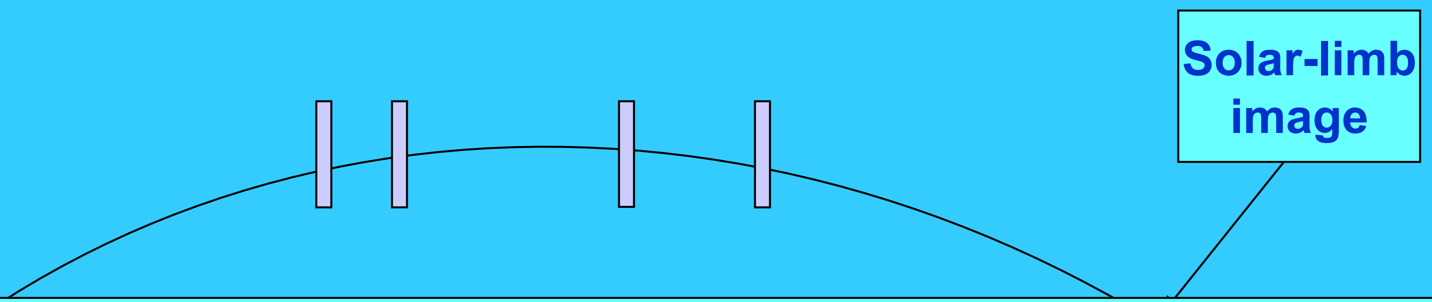
h_{max}=1146m

D_T=1m





Slits with non-redondant angular separations : multi-resolution



One has here simultaneously 6 angular baselines.
For example if the angular distance between the 2 nearest slits
is 50 arcseconds, one obtains also 100, 150, 200, 250 and 300 arcseconds.
The maximum sensing altitude varies between 4125 and 687m as Θ increases.
With the conditions of T6, L_0 being equal to 10m and D_T=1m,, **the altitude resolution [{width at C_β(b)/2} / θ] is equal to 560m when θ=50 arcseconds and to 93m when θ=300 arcseconds.**

Conclusion

- The method presented above may also be used in the case of nighttime conditions observing the the lunar limb. This profiler allows to select angular directions with high separations and thus may lead to high altitude resolution .
- It appears as complementary of an image plane profiler as, for example, the MOSP.
- A spatio-temporal analysis may be also performed.
- The effect of scintillation due to high turbulent layers must be studied theoretically and by numerical simulations for different values of θ.
- A prototype will be tested soon.